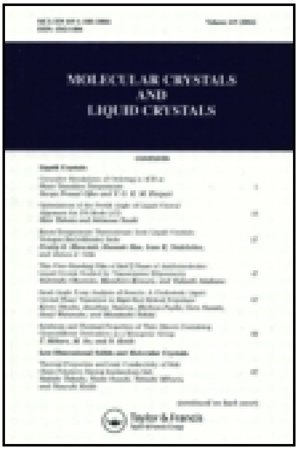

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Instability Thresholds of Nematic and Smectic-C Liquid Crystals in Elliptically Polarized Rotating Magnetic Fields

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Instability Thresholds of Nematic and Smectic-C Liquid Crystals in Elliptically Polarized Rotating Magnetic Fields†

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The behavior of nematic and smectic-C liquid crystals in elliptically polarized rotating magnetic fields is studied. The lag angle between the rotating director and the field is time-dependent and has the same rotation period as the field up to a critical frequency. The critical frequency, average torque, and the lag angle are calculated as a function of the ellipticity of the field.

It is well known that the physical properties of liquid crystals are sensitive to surface and shearing forces and to electric and magnetic fields.[1] In particular, it is known that magnetic fields give rise to a torque which tends to reorient the molecules such that they are aligned parallel to the field. (Only molecules with positive diamagnetic anisotropy will be considered here.) When the magnetic field is rotated about a fixed axis, the molecules are, in addition, subject to an effective viscous torque caused by their motion relative to the liquid. The behavior of nematic liquid crystals in circularly polarized fields rotating at fixed frequencies has been studied by Tsvetkov *et al.*,[2] Gasparoux and Prost,[3] Lee and Eringen,[4] and Leslie *et al.*[5] They found that for a normalized speed of rotation $\varepsilon_0 \leq 1$ the average direction of the molecules

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(the "director") rotates uniformly but lags the field by an angle γ_0 whose magnitude is given by

$$\sin 2\gamma_0 = \frac{\Omega}{\Omega_c}, \qquad (\Omega \leq \Omega_c) \tag{1}$$

where $\varepsilon_0 = \Omega/\Omega_c$; Ω is the angular speed of rotation, $\Omega_c = \Delta\chi H^2/\eta$ is the critical speed, $\Delta\chi = \chi_{\parallel} - \chi_{\perp} > 0$ is the diamagnetic anisotropy, H is the magnetic field, and η is a viscosity coefficient. When $\varepsilon_0 > 1$ the predicted motion is given by

$$\tan\left(\phi - \frac{\pi}{4}\right) = \left[\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}\right]^{1/2} + \tan[(\varepsilon_0^2 - 1)^{1/2}(\tau - \tau_0)], \tag{2}$$

where $\phi(\tau)$ is the director orientation in a coordinate system fixed to the sample, τ_0 is an arbitrary constant and τ is related to the time t by $\tau = \Omega_c t$. The average angular speed of the director is now $\Omega_c(\varepsilon_0^2 - 1)^{1/2}$.

As Ω approaches Ω_c from below, it has been found[2,3,5] that the measured torque does not conform to the behavior calculated from Eq. (1). Further, when $\varepsilon_0 > 1$, the measured average torque is found to be greater than that found from Eq. (2). The reasons for these deviations are not completely understood but are believed to be associated with spatial inhomogeneity of the director. In fact, it will here be shown that there exists a range of experimental conditions in which similar behavior can occur.

Consider a nematic or smectic-C liquid crystal in an elliptically polarized rotating magnetic field. The field components are in the x-y plane and are given by

$$H_x = H\cos\Omega t, \qquad H_y = aH\sin\Omega t, \tag{3}$$

where a is related to the ellipticity e of the ellipse traced by the field vector by $a^2 = 1 - e^2$ with $0 \leq e \leq 1$.

Neglecting inertial, surface, and elastic forces, the equation of motion of a nematic or smectic-C liquid crystal with its cone axis normal to the field plane may be found using (for the nematic phase) the continuum theories of Leslie[6] and Ericksen[7] or simply by equating the viscous and magnetic torques on the director. The result is

$$\dot{\phi} = \sin 2(\varepsilon\tau - \phi) - \delta^2 \sin 2(\varepsilon\tau + \phi) - 2\delta\sin 2\phi. \tag{4}$$

The normalized time variable τ is now given by $\tau = \varepsilon_0\Omega_c t/\varepsilon$, where

$$\varepsilon = \varepsilon_0(1 + \delta)^2, \qquad \delta = \frac{1 - (1 - e^2)^{1/2}}{1 + (1 - e^2)^{1/2}}. \tag{5}$$

To first order in $\delta \ll 1$, the solution of Eq. (4) is

$$\phi = \varepsilon\tau - \gamma - \delta\sin 2(\varepsilon\tau - 2\gamma), \tag{6a}$$

with

$$\sin 2\gamma = \varepsilon. \tag{6b}$$

Equation (6) describes a dynamic director motion dependent upon the field ellipticity. That is, the director does not simply follow the field with a time independent lag angle but oscillates about an average angle $\gamma = \langle \varepsilon\tau - \phi \rangle$. This average angle is greater than that which would be obtained at the corresponding rotation speed ε_0 for the circular field case. Equation (6) is valid for all γ satisfying $\sin 2\gamma \leq 1$. Choosing the equality, the critical rotation speed ε_c is found to be

$$\varepsilon_c = 1 - 2\delta + 0(\delta^2). \tag{7}$$

For the other limiting case, with $\delta \to 1$, the director motion tends to the solution

$$\tan \phi = A \exp\left[-4\left(\tau + \frac{\sin 2\varepsilon\tau}{2\varepsilon}\right)\right], \tag{8}$$

where A is an arbitrary constant. In this case, the field is essentially along the major axis of the ellipse over most of each period and the director tends to align itself parallel to this axis regardless of its initial orientation.

Since analytic solutions of Eq. (4) could be found only for the limiting cases discussed above, calculations for arbitrary values of e were carried out numerically. Results for the average torque are shown in Figure 1 as a function of ε_0 for select values of δ. As can be seen, there exists a family of curves, each with a distinct critical rotation speed ε_c, at which an anomaly occurs in the torque. As the ellipticity of the field is increased, the average torque tends to zero ever more rapidly at rotation speeds greater than the critical one. The critical curve $\varepsilon_c(\delta)$ is given in Figure 2.

For a given ellipticity, the time-dependent behavior of the director has no instabilities below the critical rotation speed and the oscillation amplitudes are functions of δ but independent of ε_0. These amplitudes, normalized with respect to 45°, are shown in Figure 2. As noted earlier, the average lag angle is always greater than the angle which would occur in the case of a circular field at the corresponding rotation speed. This can be seen clearly in Figure 3 where the sine of twice the average lag angle is shown for various values of δ as a function of ε_0.

A study was also made of the behavior of a single domain smectic-C phase for the case in which the cone axis makes an angle α with the normal to the plane in which a *circularly* polarized field is rotating. Here, in the co-ordinate system in which the cone axis coincides with the polar axis, the field will have an oscillating component along this axis and the rotating field vector in the

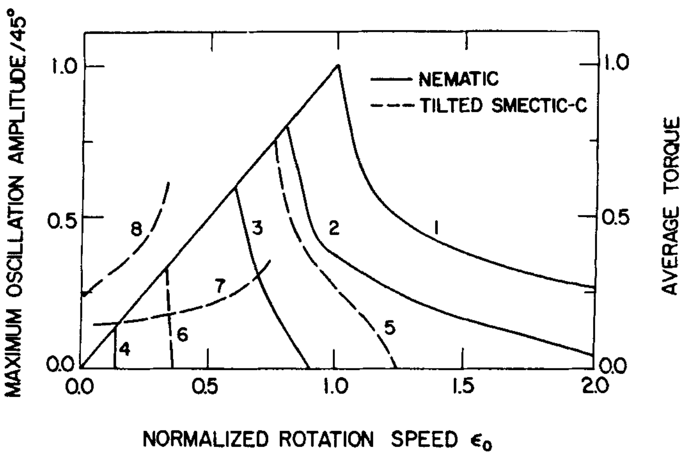

FIGURE 1 Curves 1 to 6 show the normalized average torque as a function of the normalized rotation speed ε_0. Curves 1 to 4 are valid for the nematic phase and also for the smectic-C phase when the smectic cone axis is normal to the field plane. The field ellipticity parameter δ is equal to 0.0, 0.1, 0.2, and 0.5, respectively. Curves 5 and 6 refer to the tilted smectic-C phase with $\delta = 0.025, 2\theta = 70°$ and $\delta = 0.1, 2\theta = 160°$ where 2θ is the smectic cone angle. Curves 7 and 8 give the maximum oscillation amplitude of the director on the smectic-C cone for the same parameters as 5 and 6, respectively.

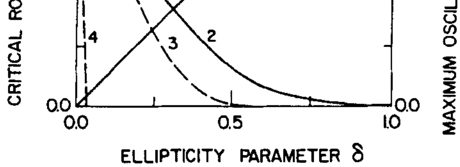

FIGURE 2 Curve 1 gives the maximum oscillation amplitude of the director as a function of the ellipticity parameter δ of the applied field for the nematic phase and also for the smectic-C phase when the smectic axis is normal to the field plane. Curves 2 to 4 give the critical frequency ε_c below which the rotating director has the same period of rotation as the field. Curve 2 applies to the same cases as curve 1 while 3 and 4 are for the tilted smectic-C phase with cone angles 2θ of 70 and 160°, respectively.

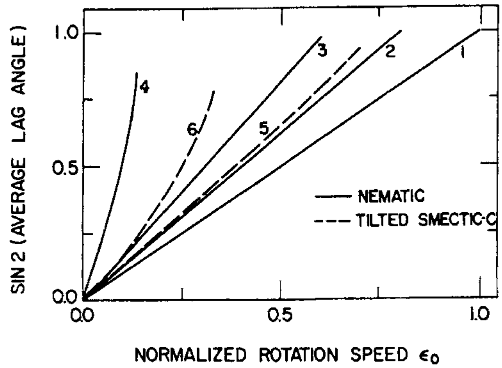

FIGURE 3 Dependence of average lag angle $\langle\gamma\rangle$ on the normalized rotation speed ε_0. Curves 1 to 4 are valid for the nematic phase and also the smectic-C phase when the smectic cone axis is normal to the field plane. The field ellipticity parameter δ is equal to 0.0, 0.1, 0.2, and 0.5, respectively. Curves 5 and 6 are for the tilted smectic-C phase with $\delta = 0.1$, $2\theta = 160°$ and $\delta = 0.025$, $2\theta = 70°$ where 2θ is the smectic cone angle.

azimuthal plane will trace out an ellipse. The equation of motion of the director is

$$\dot{\phi} = \sin 2(\varepsilon\tau - \phi) - \delta^2 \sin 2(\varepsilon\tau + \phi) - 2\delta \sin 2\phi + \delta^{1/2} \cot\theta \left\{(3 - \delta)\cos 2\left(\varepsilon\tau - \frac{\phi}{2}\right) + (1 - 3\delta)\cos 2\left(\varepsilon\tau + \frac{\phi}{2}\right) - 2(1 - \delta)\cos\phi\right\}, \tag{9}$$

where 2θ is the smectic cone angle. The motion of the director is now dependent also upon θ. The ellipticity is now given by $e = \sin\alpha$ and δ is related to e by Eq. (5).

Upon solving Eq. (7) numerically, a family of curves dependent upon the cone angle and the ellipticity was obtained. Typical torque curves for select values of δ and θ are shown in Figure 1. Note that all the normalized curves given in Figure 1 must be scaled by a factor $\Delta\chi H^2(1 - e^2)^{1/2} \cos\theta$ in order to obtain the torque component normal to the applied field. The critical rotation speed below which time-dependent solutions of the type discussed for Eq. (4) exist are shown in Figure 2. In both figures the curves for the tilted smectic-C phase are depicted by dashed lines. It is seen that the critical rotation speed decreases rapidly with decreasing cone angle and increasing tilt angle. The calculated sine of twice the average lag angle for the director moving on the tilted domain cone is shown in Figure 3.

In summary, it has been shown that nematic and smectic-C liquid crystals in the presence of elliptically polarized magnetic fields can exhibit an instability threshold when the rotation speed reaches a critical value. Results have been given for the critical frequency, average torque, and lag angle as a function of the field ellipticity. These quantities may be studied experimentally by means of either torque magnetometer[2-4] or electron resonance spectroscopy measurements.[5,8] In particular, the results presented here can be used to check the validity of the assumption that the rotational modes are spatially homogeneous under a variety of experimental conditions by varying the ellipticity of the applied field.

Acknowledgements

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